**Lecture Note: Numerical Analysis (15) Numerical Differentiation**

1. **Taylor Series Expansion and Function Derivatives Revisited**

* **Taylor series expansion of f(x+h) for a small value h around a given point x**



* **The 1st order approximation of the 1st derivative of f(x) using one of the above equation**



**Therefore, the first order numerical approximation becomes**

Forward difference formula (1st order)



Backward difference formula (1st order)



* **The 2nd order approximation of the 1st derivative of f(x) by subtracting the above equation**



**Therefore, the 2nd order numerical approximation becomes**

Central difference formula (2nd order)



* **2nd derivative (2nd order)**

Using



**Adding two equations,**



**🡪** 

1. **Higher Order Derivatives at equally spaced nodes**

* High-accuracy divided-difference formulas can be generated by including additional terms from the Taylor series expansion.



* Inclusion of the 2nd derivative term has improved the accuracy to *O(h2).*
* Similar improved versions can be developed for the backward and centered formulas as well as for the approximations of the higher derivatives.

1. **Higher Order Function Derivatives at unequally spaced nodes**

* Using the divided difference interpolating polynomial



0th order: 

1st order: 

2nd order: 

…….

nth order: 

* 1st derivatives



At ,



At ,



* 2nd derivatives



* Etc

**🡪If we apply the equal spacing condition to above relations, the higher order function derivatives can be obtained for equally spaced data**